Mathematical Modeling in Social Network Analysis: Using TOPSIS to Find Node Influences in a Social Network

Fox, William P.; Everton, Sean F.

Fox, William P., and Sean F. Everton. 2013. "Mathematical Modeling in Social Network Analysis: Using TOPSIS to Find Node Influences in a Social Network." Journal of Mathematics and System Science 3:531-41. http://www.davidpublishing.com/show.html?14534
http://hdl.handle.net/10945/53057

Journal of Mathematics and System Science 3 (2013) 531-541

Mathematical Modeling in Social Network Analysis: Using TOPSIS to Find Node Influences in a Social Network

William P. Fox and Sean F. Everton

Department of Defense Analysis, Naval Postgraduate School, Monterey, CA 93943, USA

Received: June 13, 2013 / Accepted: July 16, 2013 / Published: Octorber 25, 2013.

Abstract: In a social network analysis the output provided includes many measures and metrics. For each of these measures and metric, the output provides the ability to obtain a rank ordering of the nodes in terms of these measures. We might use this information in decision making concerning disrupting or deceiving a given network. All is fine when all the measures indicate the same node as the key or influential node. What happens when the measures indicate different key nodes? Our goal in this paper is to explore two methodologies to identify the key players or nodes in a given network. We apply TOPSIS to analyze these outputs to find the most influential nodes as a function of the decision makers' inputs as a process to consider both subjective and objectives inputs through pairwise comparison matrices. We illustrate our results using two common networks from the literature: the Kite network and the Information flow network from Knoke and Wood. We discuss some basic sensitivity analysis can may be applied to the methods. We find the use of TOPSIS as a flexible method to weight the criterion based upon the decision makers' inputs or the topology of the network.

Keywords: Social network analysis, multi-attribute decision making, Analytical hierarchy process (AHP), weighted criterion, TOPSIS, node influence

Nomenclature

$(x_{ij})_{m \times n}$	Matrix of values for alternatives by criterion
$(r_{ij})_{m \times n}$	Matrix of normalized values for alternatives by criterion
$(t_{ij})_{m \times n}$	Matrix of weighted normalized values for alternatives by criterion
A_w	Worst solution in the column
A_b	Best solution in the column
d_{ib}	L2 distance between the target and best solution
d_{iw}	L2 distance between the target and worst solution
s_{iw}	Ratio similarity to the ideal worst solution
s_{ib}	Ratio similarity to the ideal best solution
C	Final ranking

1. Introduction to Social Network Analysis

Social network analysis (SNA) is the methodical analysis of social networks in general and dark networks in particular [1, 2]. Social network analysis is a collection of theories and methods that assumes that the behavior of actors (individuals, groups, organizations, etc.) is profoundly affected by their ties to others and the networks in which they are embedded. Rather than viewing actors as automatons unaffected by those around them, SNA assumes that interaction patterns affect what actors say, do, and believe. Networks contain *nodes* (representing individual actors or entities within the network) and *edges and arcs* (representing relationships between the individuals, such as friendship, kinship, organizational position, sexual relationships, communications, tweets, Facebook friendships, etc.). These networks are often depicted in two formats: graphically or as a matrix. We might call the graph a social network diagram, where nodes are represented as *points* or *circles* and *arcs* are

Corresponding author: William P. Fox, professor, research filed: applied mathematical modeling for decision making. Email: wpfox@nps.edu.

represented as lines that interconnect the nodes.

We will provide a little background of social network analysis here. More precisely, we introduce some of the more common measures and their definitions that are used for exploratory SNA of networks. In this paper we assume we are only looking for the powerful and influential players in a network.

There are a multitude of measures (metrics) that are found in most SNA software. We begin by defining a few metric terms or measures in social network analysis that we use.

Betweenness

Betweenness is a measure of the extent to which a node lies on the shortest path between other nodes in the network. This measure takes into account the connectivity of the node's neighbors, giving a higher value for nodes which bridge clusters. The measure reflects the number of people who a person is connecting indirectly through their direct links.

Bridge

An edge is said to be a bridge if deleting it would cause its endpoints to lie in different components of a graph.

Centrality

Centrality is the measure which gives a rough indication of the social power of a node based on how well they "connect" the network. "Betweenness," "Closeness," "Degree," and “Eigenvector” are all measures of centrality.

Centralization

Centralization is the difference between the numbers of links for each node divided by maximum possible sum of differences. A centralized network will have many of its links dispersed around one or a few nodes, while a decentralized network is one in which there is little variation between the numbers of links each node possesses.

Closeness

Closeness is the degree an individual is near all other individuals in a network (directly or indirectly). It reflects the ability to access information through the "grapevine" of network members. Thus, closeness is the inverse of the sum of the shortest distances between each individual and every other person in the network. The shortest path may also be known as the "geodesic distance."

Degree

Degree is the count of the number of ties to other players in the network.

Density

Density is a measure of network cohesion that is equal to the actual number of ties in a network divided by the total possible number of ties, which means that density scores range from 0.0 to 1.0.

Eigenvector Centrality

Eigenvector centrality is a variation on degree centrality in that assumes that ties to central actors are more important than ties to peripheral actors and thus weights an actor’s summed connections to others by their centrality scores. Google’s Pagerank score is a variation on eigenvector centrality.

2. Examples of Metrics for Influential Player in Networks

2.1 Example 1. The Kite Network

We begin looking at a classic network from SNA literature. We look at the "Kite Network" (see Fig. 1), which was developed by David Krackhardt [3], a leading social network analyst. The nodes are connected by some sort of relational tie between the actors. For example, two nodes are connected if they regularly talk to each other or interact in some way. So, if Tom regularly interacts with Susan but not with Fred, Tom and Susan are connected, but there is no link drawn between Tom and Fred. This network is useful because it effectively demonstrates the distinction between the three most popular individual centrality measures that might indicate an influential node: *Degree Centrality, Betweenness Centrality,* and *Closeness Centrality*.

2.1.1 Degree Centrality

Social network researchers measure network activity

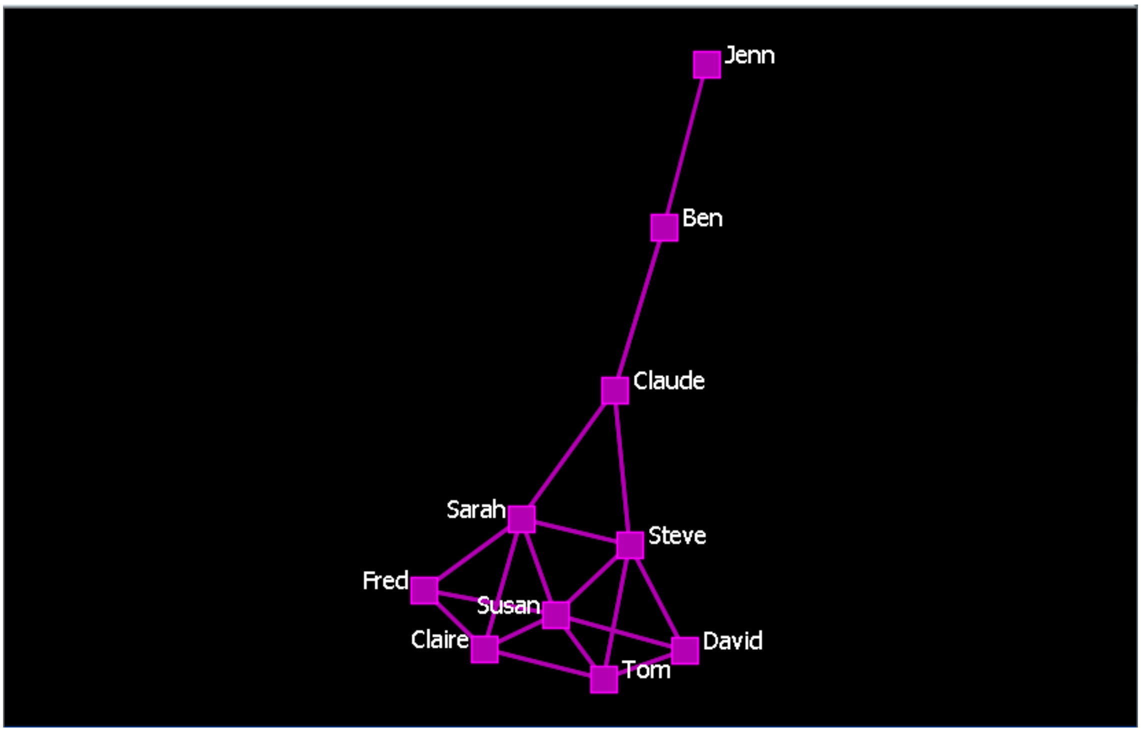

Fig. 1 'Kite Network" from ORA.

for a node by using the concept of degrees -- the number of direct connections a node has. For each member of the network, we find the number of connections to other members:

Fred 3, Claire 4, Tom 4, David 3, Steven 5, Susan 6, Sarah 5, Claudia 3, Ben 1, and Jennifer 1.

In the Kite network, Susan has the most direct connections (6) in the network, making hers the most active node in the network. She is a 'connector' or 'hub' in this network. It is often assumed that in personal networks "the more connections, the better," but this is not always so. What really matters is to where those connections lead -- and how they connect the otherwise unconnected! Here Susan has connections only to others in her immediate cluster -- her clique. She connects only those who are already connected to each other.

2.1.2 Betweenness Centrality

While Susan has many direct ties, Claudia has few – less than the average in the network – 3 as compared to the average of 3.5. Yet, in many ways, she has one of the best locations in the network -- she is *between* two important constituencies. She is in a position to play a 'brokerage' role in the network. The good news is that she plays a powerful role in the network; the bad news is that she is a single point of failure. Without her, Ben and Jennifer would be cut off from information and knowledge in Susan's cluster. A node with high *betweenness* has great influence over what flows -- and does not flow -- through a network. Claudia may *control* the outcomes in a network.

2.1.3 Closeness Centrality

Sarah and Steven have fewer connections than Susan, yet the pattern of their direct and indirect ties allow them to access all the nodes in the network more quickly than anyone else. They have the *shortest paths* to all others – in terms of path length, they are, on average, closer to everyone else. They are in an excellent position to monitor the information flow in the network -- they have the best visibility into what is happening in the network.

In summary from these three found measures we found Susan was most important from degree centrality, Claudia when we consider between centrality, and

Sarah & Steven tie in closeness centrality. So who is the most powerful and influential person in this network? We will provide a model to examine this issue.

Example 2. Information Flow Network

In 1978, Knoke & Wood [4] collected data from workers at 95 organizations in Indianapolis. Respondents indicated with which other organizations their own organization had any of 13 different types of relationships. Knoke and Kuklinski [5] selected a subset of 10 organizations and two relationships, money, and information. We will examine only the information exchange in this example. The value "1" implies there is a relationship/connection and "0" there is not a relationship/connection. The network matrix and diagram are shown in Fig. 2.

We use the matrix and figure 2 to conduct some social network analysis. We examine this network and make some useful observations about the network and the players in the network. We use Organizational Risk Analyzer (ORA) [6] to analyze the network and obtain some important measures. We begin by calculating the network density, which as defined above, equals the number of connections divided by the total number of possible connections. These

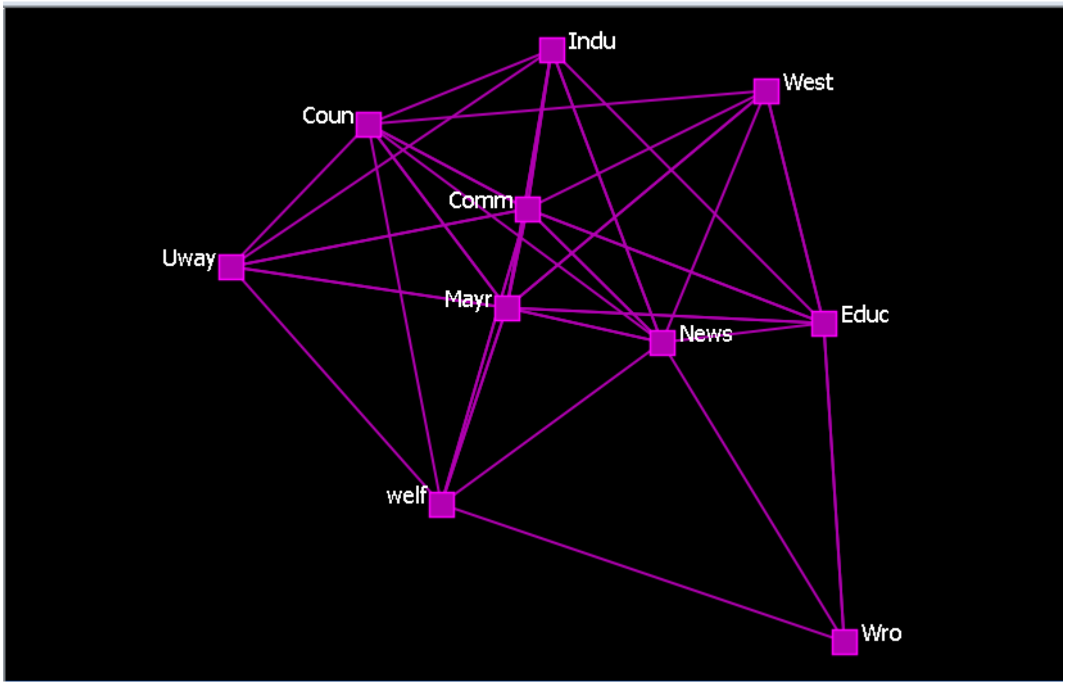

Fig. 2 Information Exchange from ORA.

Information Exchange Matrix

	COUN	COMM	EDUC	INDU	MAYR	WRO	NEWS	UWAY	WELF	WEST
1	0	1	0	0	1	0	1	0	1	0
2	1	0	1	1	1	0	1	1	1	0
3	0	1	0	1	1	1	1	0	0	1
4	1	1	0	0	1	0	1	0	0	0
5	1	1	1	1	0	0	1	1	1	1
6	0	0	1	0	0	0	1	0	1	0
7	0	1	0	1	1	0	0	0	0	0
8	1	1	0	1	1	0	1	0	1	0
9	0	1	0	0	1	0	1	0	0	0
10	1	1	1	0	1	0	1	0	0	0

connections are lines in our network.

Density = #lines/(# possible lines) = 30/(45)= 0.6667

Observations: The literature states the maximum density is 1. Our density is greater than 50%.

In the social network analysis multiple measures are calculated and analysis made. We briefly summarize these results.

Degree: Players #5 and #2 have the greatest out-degrees and might be regarded as the most influential. Players #5 and #2 are joined by #7 (the newspaper) when we examine in-degree. That other organizations share information with these three would seem to indicate a desire on the part of others to exert influence.

Path distances: Since the information network is directed, separate closeness and farness can be computed for sending and receiving. We find that player #6 has the largest sum of geodesic distances from other players and to other players.

Closeness An index of the "reach distance" from each player to (or from) all others is calculated. Here, the maximum score (equal to the number of nodes) is achieved when every other node is one-step from ego. The reach closeness sum becomes less as players are two steps, three steps, and so on (weights of 1/2, 1/3, etc.). These scores are then expressed in "normed" form by dividing by the largest observed reach value. The two tables are quite easy to interpret. The first of these shows what proportion of other nodes can be reached from each player at one, two, and three steps (in our example, all others are reachable in three steps or less). The last table shows what proportions of others can reach ego at one, two, and three steps. Note that everyone can contact the newspaper (player #7) in one step.

The next few measures are performed with specialized social network software.

Eigenvector: Next, we turn our attention to the scores of each of the cases on the 1st eigenvector. Higher scores indicate that players are "more central" to the main pattern of distances among all of the players, lower values indicate that players are more peripheral. The results are very similar to those for our earlier analysis of closeness centrality, with players #7, #5, and #2 being most central, and players #6 being most peripheral. Usually the eigenvalue approach will do what it is supposed to do: give us a "cleaned-up" version of the closeness centrality measures, as it does here.

Betweenness: Players #2, #3, and #5 appear to be relatively a good bit more powerful than others by this measure. Clearly, there is a structural basis for these players to perceive that they are "different" from others in the population. Indeed, it would not be surprising if these three players saw themselves as the movers-and-shakers, and the deal-makers that made things happen. In this sense, even though there is not very much betweenness power in the system, it could be important for group formation and stratification.

2.2.1 Information Network Summary

Social network analysis methods provide some useful tools for addressing one of the most important (but also one of the most complex and difficult) aspects of social structure: the sources and distribution of power. The network perspective suggests that the power of individual players is not an individual attribute but arises from their relations with others. Whole social structures may also be seen as displaying high levels or low levels of power as a result of variations in the patterns of ties among players. And, the degree of inequality or concentration of power in a population may be indexed.

2.2.2 Power in a Network

Power arises from occupying advantageous positions in networks of relations. Three basic sources of advantage are high degree, high closeness, and high betweenness. In simple structures (such as the star, circle, or line), these advantages tend to co-vary. In more complex and larger networks, there can be considerable disjuncture between these characteristics of a position-- so that a player may be located in a position that is advantageous in some ways, and disadvantageous in others.

We have reviewed three basic approaches to the "centrality" of individuals' positions, and some elaborations on each of the three main ideas of degree, closeness, and betweenness. This review is not exhaustive. The question of how structural position confers power remains a topic of active research and considerable debate. As you can see, different definitions and measures can capture different ideas about where power comes from, and can result in some rather different insights about social structures.

In the information exchange network, we find different key players depending on which metric we examine. We will present some methodologies and models to help access the "key" player modeling across all SNA metrics.

3. Methodologies to find key players across many metrics: application of TOPSIS Technique of Order Preference by Similarity to the ideal Solution (TOPSIS)

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a multi-criteria decision analysis method, which was originally developed in a dissertation by Hwang and Yoon in 1981 and then published [7]. It has been further development by Yoon [8], and Hwang, Lai and Liu [9]. TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution. It is a method of compensatory aggregation that compares a set of alternatives by identifying weights for each criterion, normalizing the scores for each criterion and calculating the geometric distance between each alternative and the ideal alternative, which is the best score in each criterion. An assumption of TOPSIS is that the criteria are monotonically increasing or decreasing. Normalization is usually required as the parameters or criteria are often of incompatible dimensions in multi-criteria problems. Compensatory methods such as TOPSIS allow trade-offs between criteria, where a poor result in one criterion can be negated by a good result in another criterion. This provides a more realistic form of modeling than non-compensatory methods, which include or exclude alternative solutions based on hard cut-offs.

3.1 TOPSIS Background

We only desire to briefly discuss the elements in the framework of TOPSIS. TOPSIS can be described as a method to decompose a problem into sub-problems. In most decision, the decision maker has a choice among several to many alternatives. Each alternative has a set of attributes or characteristics that can be measured, either subjectively or objectively. The attribute elements of the hierarchal process can relate to any aspect of the decision problem—tangible or intangible, carefully measured or roughly estimated, well- or poorly-understood—anything at all that applies to the decision at hand.

The TOPSIS process is carried out as follows:

Step 1 Create an evaluation matrix consisting of m alternatives and n criteria, with the intersection of each alternative and criteria given as x_{ij}, giving us a matrix $(X_{ij})_{mxn}$.

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} \begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & \cdot & \cdot & \cdot & x_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdot & \cdot & \cdot & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdot & \cdot & \cdot & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdot & \cdot & \cdot & x_{3n} \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ x_{m1} & x_{m2} & x_{m3} & \cdot & \cdot & \cdot & x_{mn} \end{bmatrix} \end{array}$$

Step 2 The matrix shown as D above then normalized to form the matrix $R=(R_{ij})_{mxn}$, using the normalization method

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}}$$

for *i=1,2…,m; j= 1,2,…n*

Step 3 Calculate the weighted normalized decision matrix. First we need the weights. Weights can come from either the decision maker or by computation.

Step 3 a. Use either the decision maker's weights for the attributes $x_1, x_2, .. x_n$ or compute the weights through the use Saaty's [10] AHP's decision maker weights method to obtain the weights as the eigenvector to the attributes versus attribute pair-wise comparison matrix.

$$\sum_{j=1}^{n} w_j = 1$$

The sum of the weights over all attributes must equal 1 regardless of the method used.

Step 3b. Multiply the weights to each of the column entries in the matrix from *Step 2* to obtain the matrix, *T*.

$$T = (t_{ij})_{m\ x\ n} = (w_j r_{ij})_{m\ x\ n}, i = 1,2, \dots, m$$

Step 4 Determine the worst alternative (A_w) and the best alternative (A_b) : Examine each attribute's column and select the largest and smallest values appropriately. If the values imply larger is better (profit) then the best alternatives are the largest values and if the values imply smaller is better (such as cost) then the best alternative is the smallest value.

$$A_w = \{\langle \max(t_{ij} \,|i = 1,2, \dots, m|j \in J_-\rangle, \langle \min(t_{ij} \,|i = 1,2, \dots, m)|j \in J_+\rangle\} \equiv \{t_{wj} \,|j = 1,2, \dots, n\},$$
$$A_{wb} = \{\langle \min(t_{ij} \,|i = 1,2, \dots, m|j \in J_-\rangle, \langle \max(t_{ij} \,|i = 1,2, \dots, m)|j \in J_+\rangle\} \equiv \{t_{bj} \,|j = 1,2, \dots, n\},$$

where,

$J_+ = \{j = 1,2, \dots n|j)$ associated with the criteria having a positive impact, and

$J_- = \{j = 1,2, \dots n|j)$ associated with the criteria having a negative impact.

We suggest that if possible make all entry values in terms of positive impacts.

Step 5 Calculate the L2-distance between the target alternative *i* and the worst condition A_w

$$d_{iw} = \sqrt{\textstyle\sum_{j=1}^{n} (t_{ij} - t_{wj})^2}, i=1,2,\dots,m$$

and the distance between the alternative i and the best condition A_b

$$d_{ib} = \sqrt{\textstyle\sum_{j=1}^{n} (t_{ij} - t_{bj})^2}, i=1,2,\dots,m$$

where d_{iw} and d_{ib} are L2-norm distances from the target alternative i to the worst and best conditions, respectively.

Step 6 Calculate the similarity to the worst condition:

$$s_{iw} = \frac{d_{iw}}{(d_{iw} + d_{ib})}, 0 \le s_{iw} \le 1, i = 1,2, \dots, m$$

$S_{iw}=1$ if and only if the alternative solution has the worst condition; and

$S_{iw}=0$ if and only if the alternative solution has the best condition.

Step 7 Rank the alternatives according to their value from S_{iw} $(i=1,2,\dots,m)$.

3.2 Normalization

Two methods of normalization that have been used to deal with incongruous criteria dimensions are linear normalization and vector normalization.

Linear normalization can be calculated as in *Step 2* of the TOPSIS process above. Vector normalization was incorporated with the original development of the TOPSIS method (see [8]), and is calculated using the following formula:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}} \text{ for } i=1,2\dots,m; j= 1,2,\dots n$$

In using vector normalization, the non-linear distances between single dimension scores and ratios should produce smoother trade-offs [11].

Let's explore two options for the weights in Step 3. First, the decision maker might actually have a weighting scheme that they want the analyst to use. In not, we suggest using Saaty's 9-Point pair-wise method developed for the Analytical Hierarchy Process (AHP) [see 10]. We briefly describe this pair-wise method to obtain weights.

We build a numerical representation using a 1-9 point scale in a pairwise comparison for the attributes criterion and the alternatives. The goal is to obtain a set of eigenvectors of the system that measures the importance with respect to the criterion. The resulting eigenvectors are the weights. We can put these values into a matrix or table based on the following:

Intensity of Importance in Pair-wise Comparisons	Definition
1	Equal Importance
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme Importance
2,4,6,8	For comparing between the above
Reciprocals of above	In comparison of elements *i* and *j* if *I* is 3 compared to *j*, then j is 1/3 compared to *i*.
Rational	Force consistency; measure values available

Several method exists to obtain these eigenvectors. One uses discrete dynamical systems. To gain some additional background of discrete dynamical systems [12,13].

Objective Statement ← This is the decision desired

Alternatives: $1, 2, 3, \ldots, n$

For each of the alternatives there are attributes to compare.

Attributes: $a_1, a_2, \ldots, a_m$

Once the hierarchy is built, the decision maker(s) systematically evaluate its various elements pairwise (by comparing them to one another two at a time), with respect to their impact on an element above them in the hierarchy. In making the comparisons, the decision makers can use concrete data about the elements, but they typically use their judgments about the elements' relative meaning and importance. It is the essence of the TOPSIS that human judgments, and not just the underlying information, can be used in performing the evaluations.

TOPSIS converts these evaluations to numerical values that can be processed and compared over the entire range of the problem. A numerical weight or priority is derived for each element of the hierarchy, allowing diverse and often incommensurable elements to be compared to one another in a rational and consistent way. This capability distinguishes the TOPSIS from other decision making techniques.

In the final step of the process, numerical priorities or ranking are calculated for each of the decision alternatives. These numbers represent the alternatives' relative ability to achieve the decision goal, so they allow a straightforward consideration of the various courses of action.

3.3 Uses and applications

While it can be used by individuals working on straightforward decisions, TOPSIS is most useful where teams of people are working on complex problems, especially those with high stakes, involving human perceptions and judgments, whose resolutions have long-term repercussions. It has unique advantages when important elements of the decision are difficult to quantify or compare, or where communication among team members is impeded by their different specializations, terminologies, or perspectives.

Decision situations to which the TOPSIS might be applied include:

- Choice: The selection of one alternative from a given set of alternatives, usually where there are multiple decision criteria involved.
- Ranking: Putting a set of alternatives in order from most to least desirable
- Prioritization: Determining the relative merit of members of a set of alternatives, as opposed to selecting a single one or merely ranking them
- Resource allocation: Apportioning resources among a set of alternatives
- Benchmarking: Comparing the processes in one's own organization with those of other best-of-breed organizations
- Quality management: Dealing with the multidimensional aspects of quality and quality improvement

• Conflict resolution: Settling disputes between parties with apparently incompatible goals or positions

3.4 Applications of TOPSIS to find influences on a network

We will illustrate TOPSIS on two examples and then compare the results to other MADM methods: Data Envelopment Analysis and Analytical Hierarchy Process.

3.4.1 Kite Network Analysis

Now we assume all we have are the outputs from ORA, Table 1.

We use the decision weights from AHP (unless a decision maker gives us their own weights) and find the eigenvectors for our eight metrics as:

w1	0.034486
w2	0.037178
w3	0.045778
w4	0.398079
w5	0.055033
w6	0.086323
w7	0.135133
w8	0.207991

We take the metrics from ORA and perform steps 2-7 of TOPSIS.

We rank order the final output from TOPSIS as shown in the last column above. We interpret the results as follows: The key node is *Susan* followed by *Steven, Sarah,* and *Claire.*

3.5 Knoke Network Analysis

We obtain the weights using the eigenvector method. The weights are:

w1	0.034486
w2	0.037178
w3	0.045778
w4	0.398079
w5	0.055033
w6	0.086323
w7	0.135133
w8	0.207991

We perform the steps in TOPSIS and obtain:

We interpret the output from TOPSIS with a rank ordering as shown in the last column above with *Mayr* as the most influential node ranked number one.

3.6 Summary and Comparisons

We have also used the two other MADM methods to rank order our nodes in previous work: DEA [14, 15, 16, 17, 18] and AHP. When we applied data envelopment analysis and AHP to compare to TOPSIS, we obtained the results displayed in Table 2 for the Kite network and Table3 for Knoke network.

3.7 Sensitivity Analysis

From DEA we can examine the reduced costs to obtain information concerning sensitivity analysis in a same fashion as normal linear programming. In both

Table 1 Summary of ORA's output for Kite Network.

	IN	OUT	Eigen	EigenL	Close	IN-Close	Betwn	INF Centr
Tom	0.4	0.4	0.46	0.296	0.357	0.357	0.019	0.111
Claire	0.4	0.4	0.46	0.296	0.357	0.357	0.019	0.109
Fred	0.3	0.3	0.377	0.243	0.345	0.345	0	0.098
Sarah	0.5	0.4	0.553	0.355	0.357	0.4	0.102	0.113
Susan	0.6	0.7	0.704	0.452	0.435	0.385	0.198	0.133
Steven	0.5	0.5	0.553	0.355	0.4	0.4	0.152	0.124
David	0.3	0.3	0.377	0.243	0.345	0.385	0	0.101
Claudia	0.3	0.3	0.419	0.269	0.385	0.385	0.311	0.111
Ben	0.2	0.2	0.097	0.062	0.313	0.313	0.178	0.062
Jennifer	0.1	0.1	0.021	0.014	0.25	0.25	0	0.039

S+	S-		C		C
0.069402	0.088959	coun	0.561749	mayr	0.944978
0.030738	0.120889	comm	0.797279	comm	0.797279
0.061268	0.085842	educ	0.583525	educ	0.583525
0.10056	0.068641	indu	0.405677	coun	0.561749
0.00815	0.139978	mayr	0.944978	news	0.555554
0.144284	0.001896	wro	0.01297	uway	0.409316
0.084879	0.106098	news	0.555554	indu	0.405677
0.102997	0.071372	uway	0.409316	welf	0.366671
0.103781	0.060085	welf	0.366671	west	0.320296
0.109895	0.051786	west	0.320296	wro	0.01297

Table 2 MADM applied to Kite network.

Node	TOPSIS Value (rank)	DEA Efficiency Value (rank)	AHP Value (rank)
Susan	0.862 (1)	1 (1)	0.159 (2)
Sarah	0.675 (3)	0.786 (2)	0.113 (4)
Steven	0.721 (2)	0.786 (2)	0.133 (3)
Claire	0.649 (4)	0.653 (4)	0.076 (6)
Fred	0.446 (8)	0.653 (4)	0.061 (8)
David	0.449 (7)	0.536 (8)	0.061 (8)
Claudia	0.540 (6)	0.595 (6)	0.176 (1)
Ben	0.246 (9)	0.138 (9)	0.109 (5)
Jennifer	0 (10)	0.030 (10)	0.036 (10)
Tom	0.542 (5)	0.553 (7)	0.076 (6)

Table 3 MADM applied to Knoke Information Exchange Network.

Node	TOPSIS Value (rank)	DEA Efficiency Value (rank)	AHP Value (rank)
Majr	0.945 (1)	1 (1)	0.171 (1)
Comm	0.798 (2)	0.689 (2)	0.150 (2)
Educ	0.584 (3)	0.653 (3)	0.118 (3)
News	0.556 (5)	0.153 (4)	0.111 (4)
Welf	0.367 (8)	0.069 (5)	0.080 (8)
Indu	0.406 (7)	0.0044 (6)	0.0184 (6)
West	0.320 (9)	0.0040 (7)	0.0734 (9)
Wro	0.013 (10)	0.020 (8)	0.045 (10)
Coun	0.562 (4)	0.020 (8)	0.087 (5)
Uway	0.409 (6)	0.00 (10)	0.081 (7)

TOPSIS and AHP we need to use a controlled “trial and error” method to find the impact of the decision maker on the criterion weights and thus, the effect of changes in the criterion weights to the altering of the ranks. We revisit the Kite network. Our decision maker has changed their pairwise comparison of the eight criterion weights. With these new criterion weights provide a new ranking ordering with Susan as the most influential node as opposed the Claudia before:

Susan	0.142161	1
Steven	0.124408	3
Sarah	0.117025	4
Tom	0.096293	5
Claire	0.096293	5
Fred	0.084394	7
David	0.084394	7
Claudia	0.125052	2
Ben	0.082076	9
Jennifer	0.047903	10

Continued trial and error can eventually find the break-point that causes the change in rank orderings.

4. Conclusion

We have provided a TOPSIS approach to ranking influential nodes (players) in a given social network. We have illustrated TOPSIS through two separate examples, the Kite and Information Exchange networks. We compared the results to two other MADM methods; DEA and AHP. We believe that the incorporation of decision maker weights with the metrics of a social network is invaluable to analysis of *key* and *influential* players.

Acknowledgement

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. This work reflects only the authors' work and not the work of the Department of Defense, the Department of the Navy, or the Naval Postgraduate School.

References

[1] S. F. Everton. Disrupting dark networks. Cambridge University Press. Cambridge, UK and New York, USA. 2012.

[2] N. Roberts, N. & S. Everton. Strategies for combating dark networks, Journal of Social Structure, 12 (2011), 1-32.

[3] D. Krackhardt, D. Assessing the political landscape: Structure, cognition, and power in organizations. Admin. Science Quarterly, 35 (1990), 342-369.

[4] D. Knoke and J. Wood. Organized for action: Commitment in voluntary associations. Rutgers University Press. New Brunwick, NJ. 1981.

[5] D. Knoke and J. Kuklinski. Network analysis. Sage Publishers. Beverly Hills, CA (1982).

[6] K. M. Carley, 2001-2011. Organizational Risk Analyzer (ORA). Pittsburgh, PA: Center for Computational Analysis of Social and Organizational Systems (CASOS): Carnegie Mellon University, USA.

[7] C. L. Hwan and K. Yoon. Multiple attribute decision making: Methods and applications. New York: Springer-Verlag. 1981.

[8] K. Yoon, K. A reconciliation among discrete compromise situations. Journal of Operational Research Society 38, (1987) 277–286.

[9] C. L. Hwang, Y, Lai, and T.Y. Liu. A new approach for multiple objective decision making. Computers and Operational Research 20 (1993) 889–899

[10] T. Satty. The analytical hierarchy process,. McGraw Hill, United States, 1980.

[11] I. B. Huang, J. Keisler, and I. Linkov. Multi-criteria decision analysis in environmental science: Ten years of applications and trends. Science of the Total Environment 409 (2011) 3578–3594

[12] W. P. Fox. Mathematical modeling of the analytical hierarchy process using discrete dynamical systems in decision analysis, Computers in Education Journal, July-Sept. (2012) 27-34.

[13] F. Giordano, W. Fox, and S. Horton. A first course in mathematical modeling, Brooks-Cole Publishers, Boston, MA. 2008

[14] A. Charnes , W. Cooper, and E. Rhodes. Measuring the efficiency of decision making units. European Journal of Operations Research, 2 (1978),429-444.

[15] W. Cooper, L. Seiford, and K. Tone. Data envelopment analysis. Kluwer Academic Press. London, UK. 2000.

[16] G. Zhenhua. The application of DEA/AHP method to supplier selection. 2009 International Conference on Information Management, Innovation Management and Industrial Engineering, (2009) 449-451.

[17] E. Thanassoulis. Introduction to the theory and application of data envelopment analysis-A foundation text with integrated software. Kluwer Academic Press. London, UK 2011.

[18] W. Winston, W. (1995) Introduction to mathematical programming. Duxbury Press, Belmont, CA., 322-325, 1995.